4. Two antibiotics are available as treatment for a common ear infection in children.
   - Antibiotic A is known to effectively cure the infection 60 percent of the time. Treatment with antibiotic A costs $50.
   - Antibiotic B is known to effectively cure the infection 90 percent of the time. Treatment with antibiotic B costs $80.

The antibiotics work independently of one another. Both antibiotics can be safely administered to children. A health insurance company intends to recommend one of the following two plans of treatment for children with this ear infection.
   - Plan I: Treat with antibiotic A first. If it is not effective, then treat with antibiotic B.
   - Plan II: Treat with antibiotic B first. If it is not effective, then treat with antibiotic A.

(a) If a doctor treats a child with an ear infection using plan I, what is the probability that the child will be cured?
   If a doctor treats a child with an ear infection using plan II, what is the probability that the child will be cured?

(b) Compute the expected cost per child when plan I is used for treatment.
   Compute the expected cost per child when plan II is used for treatment.

(c) Based on the results in parts (a) and (b), which plan would you recommend?
   Explain your recommendation.
Solution

Part (a):

Let A be the event “antibiotic A works.”
Let B be the event “antibiotic B works.”

The probability that a child will be cured with Plan I is:

\[ P(\text{Cure I}) = P(A) + P(\text{not } A) \times P(B) \]
\[ = 0.6 + (0.4 \times 0.9) \]
\[ = 0.96 \]

The probability that a child will be cured with Plan II is:

\[ P(\text{Cure II}) = P(B) + P(\text{not } B) \times P(A) \]
\[ = 0.9 + (0.1 \times 0.6) \]
\[ = 0.96 \]

Part (b):

Treatment with antibiotic A costs $50, and treatment with antibiotic B costs $80.

The expected cost per child when Plan I is used for treatment is:

\[ E(\text{Cost I}) = 50 \times 0.6 + 130 \times 0.4 \]
\[ = 30 + 52 \]
\[ = 82 \]

The expected cost per child when Plan II is used for treatment is:

\[ E(\text{Cost II}) = 80 \times 0.9 + 130 \times 0.1 \]
\[ = 72 + 13 \]
\[ = 85 \]

Part (c):

Since the probability that a child will be cured is the same under either plan, some other criterion must be used to make a recommendation. From a financial point of view, Plan I should be recommended because the expected cost per child is less than Plan II.